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A SIMPLIFIED QUANTUM GRAVITATIONAL MODEL OF INFLATION

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ABSTRACT

Inflationary quantum gravity simplifies drastically in the leading logarithm approximation. We show that the only counterterm which contributes in this limit is the 1-loop renormalization of the cosmological constant. We go further to make a simplifying assumption about the operator dynamics at leading logarithm order. This assumption is explicitly implemented at 1- and 2-loop orders, and we describe how it can be implemented nonperturbatively. We also compute the expectation value of an invariant observable designed to quantify the quantum gravitational back-reaction on inflation. Although our dynamical assumption may not prove to be completely correct, it does have the right time dependence, it can naturally produce primordial perturbations of the right strength, and it illustrates how a rigorous application of the leading logarithm approximation might work in quantum gravity. It also serves as a partial test of the “null hypothesis” that there are no significant effects from infrared gravitons.

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1 Introduction

There are profound differences between quantum gravity with a positive cosmological constant and its flat space version . Many of these differences are associated with the phenomenon of *infrared logarithms*. These are factors of the number of e-foldings since the onset of inflation that show up in the Green's functions of theories which incorporate either gravitons or massless, minimally coupled scalars. Their physical origin is the continual creation of long wavelength gravitons and scalars during inflation, which engenders a slow growth in the corresponding average field strengths. Infrared logarithms have been seen in pure quantum gravity [1], in gravity + fermions [2], in full scalar-driven inflation [3, 4], in the scalar sector of scalar-driven inflation [5], in φ^4 theory [6], in scalar QED [7] and in Yukawa theory [8, 9].

Infrared logarithms introduce a fascinating, secular element into the usual static results of flat space quantum field theory. For example, whereas the virtual gravitons of flat space have no net effect on the propagation of a single fermion, the spin-spin coupling with the sea of infrared gravitons produced during inflation makes such a fermion behave as if its field strength were being amplified by a factor of $1/\sqrt{Z_2(t)}$ with [2]:

$$Z_2(t) = 1 - \frac{17}{4\pi} GH^2 \ln\left(\frac{a(t)}{a_{\text{on}}}\right) + O(G^2) . \quad (1)$$

Here G is the Newtonian gravitational constant, H the inflationary Hubble parameter and a_{on} represents the scale factor at the onset of inflation. The dimensionless product $GH^2 \lesssim 10^{-12}$ is bounded by the current upper limit on the tensor-to-scalar ratio [10]. Despite the minuscule coupling constant, the continued growth of the scale factor $a(t)$ over a prolonged period of inflation must eventually result in this 1-loop correction becoming order unity. Because higher loop corrections also become order unity at about the same time it is clear that perturbation theory breaks down and that one must employ a nonperturbative technique to evolve further.

The eventual breakdown of perturbation theory evident in expression (1) is a general feature of quantum field theories that exhibit infrared logarithms. Starobinskiĭ has developed a simple stochastic formalism [11] which has been proved to reproduce the leading infrared logarithms of scalar potential models at arbitrary loop order [12]. When the scalar potential is bounded below this technique can even be used to evolve past the breakdown of perturbation

theory to asymptotically late times [13]. Starobinskiĭ’s technique has recently been generalized to scalar models which involve fields that do not produce infrared logarithms such as fermions [9] and photons [14]. However, it has not yet been extended to the derivative couplings of quantum gravity.

The purpose of this paper is to explore quantum gravity at leading logarithm order.¹ We have two results, one rigorous and the other qualitative. The rigorous result is that “leading log” quantum gravity is much better behaved than full quantum gravity. The only counterterm that contributes at leading logarithm order is the 1-loop renormalization of the cosmological constant. This is derived in Section 2.

Our qualitative result consists of a simplifying dynamical assumption, the *Effective Scale Factor Approximation*, under which the leading infrared logarithms of quantum gravity can be computed and summed to give non-perturbative results. Although this assumption may not be entirely right, its qualitative features are consistent with all that is currently known about infrared logarithms, and the approximation allows us to explore how a rigorous application of Starobinskiĭ’s formalism might work in pure quantum gravity.

Both results stand on their own, however, it is interesting to view them in the context of our long-held suspicion that inflation might be driven by a positive bare cosmological constant without any scalar inflaton [17, 1]. An attractive feature of this idea is that it dispenses with the usual problem of explaining why the cosmological constant is so small by assuming that Λ is actually GUT-scale. That would start inflation in the early universe without the need to assume an unnaturally smooth initial condition. Inflation would be stopped in this model by the back-reaction from long wavelength, virtual gravitons which are continually ripped out of the vacuum by the accelerated expansion of inflation. The kinetic energy density of these gravitons is exactly diluted by the expansion of the 3-volume to produce a constant energy density. However, the next order effect, due to the gravitational interaction energy between gravitons, must slow inflation because gravity is attractive. This next order effect grows without bound, so it must eventually stop inflation if nothing else supervenes first.

One can understand the growth of back-reaction from the fact that the Newtonian potential is $-GM/R$, with the total kinetic energy of infrared gravitons growing like $M \sim a^3(t)$ and the radius growing like $R \sim a(t)$. Hence the universe must eventually fall inside its own Schwarzschild radius! That

¹For some interesting alternate approaches, see [15, 16].

estimate (of $a^2(t)$ growth for the energy density of interaction) assumes the newly created gravitons are instantly in contact with one another, whereas the self-interaction actually requires time to build up. When this causality delay is taken into account the interaction energy density grows like $-GH^6 \ln(a)$ at lowest order [18]. This slower growth means that inflation can only be checked after an *enormous* number of e-foldings: $\ln(a) \sim 1/(GH^2) \sim 10^{12}$. There seems to be no problem with such a large number of e-foldings; indeed, it can be viewed as a way of exploiting the weakness of the gravitational interaction to make inflation last a long time. We will see that it is also a way of keeping spatial variation small even over volumes as large as the currently observable universe.

Although we emphasize that the results of the present work stand on their own, they can be viewed as a partial check on the “null hypothesis” that infrared gravitons make no significant corrections to inflationary quantum gravity [19, 20]. Further, if one assumes that the spatially homogeneous, leading infrared logarithms stop inflation, then the spatially inhomogeneous corrections at the next order are suppressed by one factor of GH^2 , which is the observed strength of the spectrum of primordial density perturbations. Hence scalar density perturbations of the right strength seem to be possible without a fundamental scalar.

Section 3 motivates and defines the Effective Scale Factor Approximation. In Section 4 we compute the effective scale factor, both perturbatively using dimensional regularization at 1- and 2-loop orders and, implicitly, at arbitrary order. Section 5 sees this result used to evaluate an invariant observable which has been proposed to quantify the quantum gravitational back-reaction on inflation [21]. Our conclusions comprise Section 6.

Before closing this Introduction we should clarify the distinction between “full quantum gravity” and “quantum gravity at leading logarithm order”. Because general relativity is not perturbatively renormalizable, it cannot provide a complete theory of quantum gravity on the perturbative level. There must either be some different model, possibly not even based upon a metric field, or else quantum gravity computations are intrinsically nonperturbative. However, failing to determine *everything* is not quite the same thing as failing to determine *anything*. It is perfectly valid to employ perturbative quantum general relativity as an effective field theory to study phenomena which are driven by infrared gravitons. Further, the results so obtained cannot be changed by the still unknown, ultraviolet completion of the theory.

So we will many times derive results which take the form,

$$\left(\text{finite number}\right) \times \ln(a) + \left(\text{divergent constant}\right), \quad (2)$$

and retain the infrared logarithm while ignoring the divergent constant. This is correct because the infrared logarithm derives exclusively from long wavelength virtual gravitons, which must be reliably described by quantum general relativity. In contrast, the divergent constant originates in the ultraviolet sector, which cannot be correct, at least not on the perturbative level. Because no divergences are observed in nature we know that the ultraviolet completion of quantum gravity is somehow avoiding them. As long as the result is a finite constant, the infrared logarithm must eventually dominate at late times.

This use of a flawed (or misunderstood) formalism as an effective quantum field theory to infer valid results from the infrared has a long and distinguished history. The oldest example is the solution of the infrared problem in quantum electrodynamics by Bloch and Nordsieck [22], long before that theory's renormalizability was suspected. Weinberg [23] was able to achieve a similar resolution for quantum gravity with zero cosmological constant. The same principle was at work in the Fermi theory computation of the long range force due to loops of massless neutrinos by Feinberg and Sucher [24]. Matter which is not supersymmetric generates nonrenormalizable corrections to the graviton propagator at one loop, but this did not prevent the computation of photon, massless neutrino and massless, conformally coupled scalar loop corrections to the long range gravitational force [25]. More recently, Donoghue [26] has touched off a minor industry [27] by applying the principles of low energy effective field theory to compute graviton corrections to the long range gravitational force. Our analysis exploits the power of low energy effective field theory in the same way, differing from the previous examples only in the detail that our background geometry is locally de Sitter rather than flat.

2 Quantum Gravity at Leading Log Order

The purpose of this section is to describe the leading logarithm approximation for inflationary quantum gravity and to demonstrate that the theory in this limit is vastly better behaved than full quantum gravity. We begin by reviewing the free field expansion of quantum gravity on a locally de Sitter background. We then take note of some key facts about infrared logarithms.

Finally, we show that only the 1-loop renormalization of the cosmological constant can contribute at leading logarithm order.

2.1 The Free Field Expansion

Because we employ dimensional regularization it is necessary to work in D spacetime dimensions. With a cosmological constant Λ , the gravitational equations of motion are:²

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{1}{2}(D-2)\Lambda g_{\mu\nu} = 0 \quad . \quad (3)$$

The free field expansion of perturbative quantum gravity has much the same structure on a locally de Sitter background as on flat space. Consider the case of synchronous gauge for which the invariant element takes the form:

$$ds^2 = -dt^2 + a^2(t) \exp\left[\kappa h(t, \vec{x})\right]_{ij} dx^i dx^j \quad , \quad a(t) \equiv e^{Ht} \quad . \quad (4)$$

Here, the exponential of the graviton field $h_{ij}(t, \vec{x})$ has its usual meaning:

$$\exp\left[\kappa h\right]_{ij} \equiv \delta_{ij} + \kappa h_{ij} + \frac{1}{2}\kappa^2 h_{ik} h_{kj} + \dots \quad (5)$$

where $\kappa^2 \equiv 16\pi G$ is the loop counting parameter.

Just as in flat space we can decompose the graviton field into irreducible representations of the rotation group:

$$h_{ij} \equiv h_{ij}^{TT} + \partial_i h_j^T + \partial_j h_i^T - \frac{1}{D-2} \left[\delta_{ij} - (D-1) \frac{\partial_i \partial_j}{\nabla^2} \right] h^L + \frac{1}{D-2} \left[\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} \right] h \quad , \quad (6)$$

where the usual conditions distinguish the transverse-traceless and transverse components:

$$\partial_i h_{ij}^{TT} = h_{ii}^{TT} = \partial_i h_i^T = 0 \quad . \quad (7)$$

Like flat space, the linearized equations of motion imply:

$$h_i^T(t, \vec{x}) = h^L(t, \vec{x}) = h(t, \vec{x}) = 0 \quad , \quad (8)$$

²Hellenic indices take on spacetime values while Latin indices take on the $(D-1)$ space values. Our metric tensor has spacelike signature and our curvature tensor equals $R^\alpha_{\beta\mu\nu} \equiv \Gamma^\alpha_{\nu\beta,\mu} + \Gamma^\alpha_{\mu\rho} \Gamma^\rho_{\nu\beta} - (\mu \leftrightarrow \nu)$. The Hubble parameter is $H \equiv \sqrt{\frac{1}{D-1}\Lambda}$.

while the linearized solution for the transverse-traceless components can be expanded in spatial plane waves:

$$\chi_{ij}(t, \vec{x}) \equiv \sqrt{2} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \theta(k-H) \sum_{\lambda} \left\{ u(t, k) e^{i\vec{k}\vec{x}} \epsilon_{ij}(\vec{k}, \lambda) \alpha(\vec{k}, \lambda) + (cc) \right\} . \quad (9)$$

In (9), $u(t, k)$ are the mode functions, $\epsilon_{ij}(\vec{k}, \lambda)$ is the polarization tensor, $\alpha(\vec{k}, \lambda)$ the annihilation operator, and (cc) stands for complex conjugation. The $\frac{1}{2}(D-3)D$ creation and annihilation operators and transverse-traceless polarization tensors obey the same relations as in flat space [28, 29]:

$$\left[\alpha(\vec{k}, \lambda), \alpha^\dagger(\vec{k}', \lambda') \right] = (2\pi)^{D-1} \delta_{\lambda\lambda'} \delta^{D-1}(\vec{k} - \vec{k}') , \quad (10)$$

$$\sum_{\lambda} \epsilon_{ij}(\vec{k}, \lambda) \epsilon_{k\ell}^*(\vec{k}, \lambda) = \Pi_{i(k)}(\hat{k}) \Pi_{\ell j}(\hat{k}) - \frac{1}{D-2} \Pi_{ij}(\hat{k}) \Pi_{k\ell}(\hat{k}) , \quad (11)$$

$$\epsilon_{ii}(\vec{k}, \lambda) = k_i \epsilon_{ij}(\vec{k}, \lambda) = 0 , \quad \epsilon_{ij}(\vec{k}, \lambda) \epsilon_{ij}^*(\vec{k}, \lambda') = \delta_{\lambda\lambda'} , \quad (12)$$

where $\Pi_{ij}(\hat{k})$ is the transverse projection operator:

$$\Pi_{ij}(\hat{k}) \equiv \delta_{ij} - \hat{k}_i \hat{k}_j , \quad (13)$$

and where parenthesized indices are symmetrized.

The main difference from flat space is that the mode functions $u(t, k)$ go from simple exponentials to Hankel functions:³

$$u(t, k) = i \sqrt{\frac{\pi}{4H}} a^{-(\frac{D-1}{2})} H_{\frac{D-1}{2}}^{(1)}\left(\frac{k}{Ha}\right) . \quad (14)$$

The restriction to $k \equiv \|\vec{k}\| \geq H$ in (9) is made to avoid an infrared singularity in the free propagator [30]. The physical reason for this singularity is that no causal process would allow a local observer to prepare coherent Bunch-Davies vacuum over an infinite spatial section. Sensible physics can be regained either by employing an initial state for which the super-horizon modes are less strongly correlated [31], or else by working on a compact spatial manifold such as T^{D-1} for which there are initially no super-horizon modes [32], and then making the integral approximation to the mode sums. In both cases modes with $k < H$ are effectively absent.

³The system is assumed to be released in free Bunch-Davies vacuum at $t = 0$.

The parallel with flat space persists at higher orders. The fundamental quantity of the free field expansion is the linearized mode sum (9). All components of h_{ij} can be expressed in terms of χ_{ij} . The g_{00} and g_{0i} constraint equations determine h_i^T , h^L and h as expansions which start at second order:

$$h_i^T \sim h^L \sim h \sim \kappa \chi^2 + \kappa^2 \chi^3 + \dots \quad (15)$$

The dynamical g_{ij} equations define a related expansion for h_{ij}^{TT} which starts at first order:

$$h_{ij}^{TT} \sim \chi_{ij} + \kappa \chi^2 + \kappa^2 \chi^3 + \dots \quad (16)$$

These expansions will generally involve integrations against retarded Green's functions – which can themselves be expressed as the commutator of two free fields:

$$[{}_{ij}G_{k\ell}](x; x') = i\theta(t - t') [\chi_{ij}(x), \chi_{k\ell}(x')] \quad (17)$$

A nice diagrammatic representation for the free field expansions has been given recently by Musso [33].

The expectation value of any operator can be computed using the free field expansion. One first expands the operator in powers of the graviton field h_{ij} . Each term in this series gives its own expansion in powers of the free field χ_{ij} . One then takes the expectation value of the resulting sums and integrals of products of free fields, for which the usual reductions of free field correlators apply. In particular, the expectation value of N of the χ_{ij} 's is zero for N odd, while for N even it is the sum of the $(N - 1)!!$ distinct permutations of 2-point correlators.

2.2 Facts about Infrared Logarithms

We have already seen the crucial role of infrared logarithms for perturbative quantum gravity on a locally de Sitter background. These infrared logarithms derive from explicit factors of $\ln(a)$ ⁴ in free field correlators [34], and from integrating the retarded Green's functions (times these correlators) that arise in the free field expansion [35, 3]. Based on this understanding one can give a simple rule – valid for any interaction in any theory – for counting the maximum number of infrared logarithms that can arise for each extra power

⁴To simplify the notation we have normalized the initial value of the scale factor to be $a_{\text{on}} = 1$.

of the coupling constant. Consider an interaction with N undifferentiated massless, minimally coupled scalars or gravitons:

$$\Delta\mathcal{L} \sim (\text{coupling constant}) \times (\varphi, h_{ij})^N \times (\text{anything}) . \quad (18)$$

Here, “anything” can involve differentiated φ or h_{ij} ’s, or other fields altogether. Then, for each additional factor of the square of the coupling constant there can be at most N additional infrared logarithms [14]. A few examples from scalar models on a non-dynamical de Sitter background illustrate the rule [6, 7, 8]:

$$\Delta\mathcal{L} = -\frac{1}{4!}\lambda\varphi^4 a^D \implies \lambda \ln^2(a) , \quad (19)$$

$$\Delta\mathcal{L} = ie\varphi^* A_\mu \partial_\nu \varphi \eta^{\mu\nu} a^{D-2} \implies e^2 \ln(a) , \quad (20)$$

$$\Delta\mathcal{L} = -f\varphi\bar{\psi}\psi a^D \implies f^2 \ln(a) . \quad (21)$$

The basic interaction of the bare quantum gravitational Lagrangian has long been known for de Sitter background [36] and it gives the result [14]:

$$\Delta\mathcal{L} \sim \kappa h \partial h \partial h a^{D-2} \implies GH^2 \ln(a) . \quad (22)$$

The counting is the same for gravity + fermions [2]:

$$\Delta\mathcal{L} \sim \kappa h \bar{\psi} \partial \psi a^{D-1} \implies GH^2 \ln(a) . \quad (23)$$

Hence, the general form of the fermion field strength (1) is:

$$Z_2(t) = 1 + \sum_{\ell=1}^{\infty} (GH^2)^\ell \left\{ c_{\ell,0} [\ln(a)]^\ell + c_{\ell,1} [\ln(a)]^{\ell-1} + \dots + c_{\ell,\ell-1} \ln(a) \right\} . \quad (24)$$

The constants $c_{\ell,k}$ are pure numbers which are assumed to be of order one. The term in (24) involving $[GH^2 \ln(a)]^\ell$ is the *leading logarithm* contribution at ℓ loop order; the other terms are *subdominant logarithms*.

Quantum gravitational perturbation theory breaks down when $\ln(a) \sim 1/GH^2$, at which point the leading infrared logarithms at each loop order contribute numbers of order one. In contrast, the subleading logarithms are all suppressed by at least one factor of the very small number $GH^2 \lesssim 10^{-12}$. So it makes sense to retain only the leading infrared logarithms:

$$Z_2(t) \longrightarrow 1 + \sum_{\ell=1}^{\infty} c_{\ell,0} [GH^2 \ln(a)]^\ell . \quad (25)$$

This is known as the *leading logarithm approximation*.

It is important to note that, *the operator under study can affect the form of its leading logarithm expansion*. For example, we saw from expression (19) that one gets at most two infrared logarithms for each extra coupling constant from a $\lambda\varphi^4 a^D$ interaction. That rule only gives the maximum, and it does not specify the number of infrared logarithms in the lowest order result. Explicit computation shows that the leading logarithm expansion of the expectation value of $2n$ coincident fields is [12]:⁵

$$\begin{aligned} \langle\langle \Omega | \varphi^{2n}(x) | \Omega \rangle\rangle &= (2n-1)!! \left[\frac{H^2 \ln(a)}{4\pi^2} \right]^n \left\{ 1 - \frac{n^2+n}{2} \frac{\lambda}{36\pi^2} \ln^2(a) \right. \\ &\quad \left. + \frac{35n^4+170n^3+225n^2+74n}{280} \left[\frac{\lambda}{36\pi^2} \ln^2(a) \right]^2 - \dots \right\} . \end{aligned} \quad (26)$$

In this case the bound of $\ln^2(a)$ for each extra λ is saturated for every n , but the $\ln(a)$ dependence of the order λ^0 result depends upon n . In contrast, the expectation value of the kinetic term takes the form [6]:

$$\langle\langle \Omega | g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi | \Omega \rangle\rangle = -\frac{3H^4}{8\pi^2} \left\{ 1 + \# \lambda \ln(a) + O(\lambda^2 \ln^3(a)) \right\} , \quad (27)$$

where $\#$ is a divergent constant.⁶ Note that the derivatives in the operator under study not only preclude there being any infrared logarithms at zeroth order, they also cause loop corrections to be one fewer than the maximum given by the rule (19). So instead of the order λ correction possessing two infrared logarithms, it has only one; instead of the order λ^2 correction having four infrared logarithms, it has only three; and so on.

⁵We shall use henceforth $\langle\langle O \rangle\rangle$ to indicate that the expectation value of an arbitrary operator O is taken in the leading logarithm approximation.

⁶One can either view this kinetic contribution as part of the stress tensor, in which case it requires no composite operator renormalization but it also fails to contribute at leading logarithm order. Such divergent, subleading logarithms are automatically removed by renormalization as a consequence of the cancellation of overlapping divergences, but they can leave calculable, finite remainders. (For worked-out examples see [6].) Or one can view the coincident kinetic term as a composite operator in its own right, which of course requires composite operator renormalization to produce a finite result. For an explicit example of how even the leading logarithm contributions to such composite operators can be divergent see section 4 of [9].

2.3 Quantum Gravity

Working in the leading logarithm approximation effects a dramatic simplification in quantum gravity. Full quantum gravity is haunted by the yet unknown – and experimentally untested – nature of its ultraviolet completion. In perturbation theory this shows up as an escalating series of divergences and counterterms whose arbitrary finite parts degrade – but don’t entirely eliminate [26] – the theory’s ability to make predictions. In contrast, the leading logarithm terms of quantum gravity are largely dominated by the infrared sector, whose preservation is an essential correspondence limit of any ultraviolet completion. And the only counterterm that contributes at leading logarithm order is the 1-loop renormalization of the cosmological constant.

This crucial insight concerning counterterms derives from the rule given above. The cosmological counterterm is:

$$\Delta\mathcal{L}_1 \equiv -\frac{(D-2)\delta\Lambda}{16\pi G} \sqrt{-g} . \quad (28)$$

We can express $\delta\Lambda$ as dimensionless contributions from one loop ($\delta\Lambda_1$), two loops ($\delta\Lambda_2$) and so forth:

$$\delta\Lambda \equiv H^2 \left\{ \delta\Lambda_1 \times \kappa^2 H^2 + \delta\Lambda_2 \times (\kappa^2 H^2)^2 + \dots \right\} . \quad (29)$$

The measure factor has its standard expansion:

$$\sqrt{-g} = a^D \left\{ 1 + \frac{1}{2} \kappa h_{ii} + \frac{1}{8} \kappa^2 (h_{ii})^2 + \dots \right\} . \quad (30)$$

It follows that the cosmological counterterm has the general form:

$$\Delta\mathcal{L}_1 \sim H^4 \left\{ \delta\Lambda_1 + \delta\Lambda_2 \times \kappa^2 H^2 + \dots \right\} \times \kappa^n h^n a^D . \quad (31)$$

The 1-loop term has the same number of undifferentiated gravitons per coupling constant as the bare interaction (22), so it contributes at leading logarithm order. However, one can see that the 2-loop term – the one proportional to $\delta\Lambda_2$ – is suppressed by a factor of the minuscule parameter $\kappa^2 H^2$. It contributes to the first subleading logarithm term, and is completely absent from the leading logarithm approximation. The higher loop contributions to $\delta\Lambda$ are even more suppressed.

The higher counterterms of quantum gravity can be organized so that their graviton expansions always contain differentiated fields:

$$\Delta\mathcal{L}_2 \equiv -\frac{\delta G}{16\pi G^2} [R - (D-2)\Lambda] \sqrt{-g} \ , \quad (32)$$

$$\Delta\mathcal{L}_3 \equiv \alpha [R - D\Lambda]^2 \sqrt{-g} \ , \quad (33)$$

$$\Delta\mathcal{L}_4 \equiv \beta C^{\rho\sigma\mu\nu} C_{\rho\sigma\mu\nu} \sqrt{-g} \ , \quad (34)$$

and so on. As with $\delta\Lambda$, we can express δG in terms of dimensionless, ℓ -loop contributions δG_ℓ :

$$\delta G \equiv \kappa^2 \left\{ \delta G_1 \times \kappa^2 H^2 + \delta G_2 \times (\kappa^2 H^2)^2 + \dots \right\} . \quad (35)$$

Hence, the Newtonian constant counterterm takes the form:

$$\Delta\mathcal{L}_2 \sim \kappa^2 H^2 \left\{ \delta G_1 + \delta G_2 \times \kappa^2 H^2 + \dots \right\} \left\{ \partial h \partial h + \kappa h \partial h \partial h + \dots \right\} a^{D-2} . \quad (36)$$

When compared with the bare interaction (22) even the 1-loop contribution has an extra factor of $\kappa^2 H^2$ for each undifferentiated graviton. Therefore, the Newtonian constant counterterm makes no contribution at leading logarithm order, and the same holds for *all* higher counterterms.

The reason for this amelioration of the ultraviolet problem is that undifferentiated fields effectively lose their ultraviolet modes at leading logarithm order. One can only reach leading logarithm order if every term in the free field expansion that can contribute a factor of $\ln(a)$ actually does so. These factors of $\ln(a)$ come only from the infrared sector, that is, from $H < k < Ha(t)$ in the free field mode sum (9) [14]. For example, the expectation value of two coincident free fields is:

$$\begin{aligned} \langle \Omega | \chi_{ij}(t, \vec{x}) \chi_{k\ell}(t, \vec{x}) | \Omega \rangle &= \frac{2D(D-3)}{(D+1)(D-2)} \left[\delta_{i(k} \delta_{\ell)j} - \frac{1}{D-1} \delta_{ij} \delta_{k\ell} \right] \\ &\times \frac{1}{2^{D-2} \pi^{\frac{D-1}{2}} \Gamma(\frac{D-1}{2})} \int_H^\infty dk k^{D-2} \|u(t, k)\|^2 \ , \quad (37) \end{aligned}$$

$$\begin{aligned} &= \frac{2D(D-3)}{(D+1)(D-2)} \left[\delta_{i(k} \delta_{\ell)j} - \frac{1}{D-1} \delta_{ij} \delta_{k\ell} \right] \\ &\times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\{ (UV) + \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} 2 \ln(a) \right\} . \quad (38) \end{aligned}$$

To get the ultraviolet divergent constant (UV) requires the full range of integration and the complete form of the integrand. However, this divergent

constant is guaranteed to be of subleading logarithm order, so it holds no interest for us. To extract the finite infrared logarithm it suffices to retain only the first term in the long wavelength expansion of the mode function:

$$k^{D-2} \|u(t, k)\|^2 = \frac{\Gamma^2(\frac{D-1}{2}) 2^{D-3} H^{D-2}}{\pi k} \left\{ 1 + O\left(\frac{k^2}{H^2 a^2}\right) \right\} . \quad (39)$$

We can furthermore restrict the range of integration to just the infrared:

$$\begin{aligned} \frac{1}{2^{D-2} \pi^{\frac{D-1}{2}} \Gamma(\frac{D-1}{2})} \int_H^\infty dk k^{D-2} \|u(t, k)\|^2 &\longrightarrow \frac{\Gamma(\frac{D-1}{2}) H^{D-2}}{2\pi^{\frac{D+1}{2}}} \int_H^{Ha} \frac{dk}{k} , \quad (40) \\ &= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} 2 \ln(a) . \quad (41) \end{aligned}$$

The simplifications noted above do not apply to the correlators of differentiated free fields. Because these correlators cannot contribute infrared logarithms, the constants they contribute (times one power of a for every spatial derivative) come from all parts of the mode sum and from all terms in the mode function. It follows that *the correlators of differentiated free fields must be evaluated exactly, with the dimensional regularization in place.*

It is well to close this section with a summary of facts about infrared logarithms in quantum gravity:

- There can be at most one factor of $\ln(a)$ for each extra factor of the loop counting parameter GH^2 ;
- The operator under study controls how many infrared logarithms occur at lowest order, and it may also cause the number expected at higher orders to be fewer than the maximum just noted;
- The only counterterm that affects the leading logarithm order is the 1-loop renormalization of the cosmological constant; and
- Even at leading logarithm order the correlators of differentiated free fields must be computed exactly, with the (dimensional) regularization in place.

3 Effective Scale Factor Approximation

The purpose of this section is to explain the Effective Scale Factor Approximation. We begin by simply defining the approximation. We then give a motivation for it. Finally, we comment on the potential problem of using expectation values rather than stochastic samples, and the closely related issue of spatial inhomogeneities.

3.1 The Approximation

The dynamical variable of synchronous gauge quantum gravity is the spatial metric, $g_{ij}(t, \vec{x})$. On de Sitter background it is natural to express this as follows in terms of the graviton field $h_{ij}(t, \vec{x})$:

$$g_{ij}(t, \vec{x}) \equiv a^2(t) \exp[\kappa h(t, \vec{x})]_{ij} \quad , \quad a(t) = e^{Ht} \quad . \quad (42)$$

We have seen that the various components of the graviton field can be expanded in powers of the transverse-traceless free field $\chi_{ij}(t, \vec{x})$, which is defined by expressions (9-14). We also saw from (38) that infrared logarithms derive from the steady growth in the amplitude of χ_{ij} .

The complete expansion of h_{ij} in powers of χ_{ij} reflects the full complexity of perturbative quantum gravity, including the ultraviolet. It is neither possible to retain all this complexity, nor would it even be desirable in the absence of fully understanding the physical ultraviolet completion of the theory. What we seek instead is a simplified expansion that reproduces the leading infrared logarithms. We propose that the *Effective Scale Factor Approximation* may provide such an expansion. The basic idea is that the graviton remains transverse-traceless and free, but propagates in the background geometry of an effective scale factor $A(t)$ which is determined from the expectation value of the g_{00} gravitational constraint equation of motion. More precisely, this approximation is defined by three statements:

1. The full metric is the square of a \mathbb{C} -number, effective scale factor $A(t)$ times the exponential of a transverse-traceless mode sum $H_{ij}[A](t, \vec{x})$:

$$g_{ij}(t, \vec{x}) \longrightarrow A^2(t) \exp[\kappa H[A](t, \vec{x})]_{ij} \quad . \quad (43)$$

2. The transverse-traceless mode sum $H_{ij}[A](t, \vec{x})$ is the same as the free field $\chi_{ij}(t, \vec{x})$ with the de Sitter mode functions (14) replaced by the

functions $u[A](t, k)$ that pertain for scale factor $A(t)$ [37]:

$$H_{ij}[A](t, \vec{x}) \equiv \sqrt{2} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \theta(k-H) \sum_{\lambda} \left\{ u[A](t, k) e^{i\vec{k}\vec{x}} \epsilon_{ij}(\vec{k}, \lambda) \alpha(\vec{k}, \lambda) + (cc) \right\} . \quad (44)$$

3. The effective scale factor $A(t)$ is determined from the leading logarithm expectation value of the g_{00} equation of motion (3), including the 1-loop renormalization of the cosmological constant:

$$\langle\langle \Omega \left| R_{00} + \frac{1}{2}R - \frac{1}{2}(D-2)(\Lambda + \delta\Lambda_1\kappa^2 H^4) \right| \Omega \rangle\rangle = 0 . \quad (45)$$

Note that the expectation value of any operator in the presence of a homogeneous and isotropic state such as the vacuum must itself be homogeneous and isotropic, even if the operator is not, so (45) can indeed be regarded as an equation for $A(t)$.

3.2 Motivation

In terms of the graviton's general decomposition (6) it will be seen that the Effective Scale Factor Approximation amounts to the following assumptions about the behavior at leading log order:

$$h_{ij}^{TT}(t, \vec{x}) \longrightarrow H_{ij}[A](t, \vec{x}) , \quad (46)$$

$$h_i^T(t, \vec{x}) \longrightarrow 0 , \quad (47)$$

$$h^L(t, \vec{x}) \longrightarrow \ln \left[\frac{A^2(t)}{a^2(t)} \right] , \quad (48)$$

$$h(t, \vec{x}) \longrightarrow (D-1) \ln \left[\frac{A^2(t)}{a^2(t)} \right] . \quad (49)$$

Because we do not yet know how to derive these assumptions from the leading logarithm approximation it is best to regard them, at this stage, as being *in addition* to it. However, we suspect that a proper derivation exists and we advance the following reasons for this belief:

- A key distinction between gravity and the various scalar models [6, 7, 8] that have been studied is that a constant value of the graviton field has

no gauge invariant significance. Unlike scalar theories, gravitation cannot develop an effective potential other than the cosmological constant itself. There are certainly quantum corrections, but they must always involve derivatives, which reduces the number of infrared logarithms. This suggests that the leading logarithm theory may be “free” in some variable. A plausible candidate for this variable is $h_{ij}^{TT}(t, \vec{x})$ because it is the only component of the graviton that involves $\chi_{ij}(t, \vec{x})$ at first order.

- By rotational covariance the transverse field $h_i^T(t, \vec{x})$ must involve a spatial derivative, which should preclude this component of the graviton from ever contributing at leading logarithm order.
- Note that constraint equations determine $h^L(t, \vec{x})$ and $h(t, \vec{x})$ as functionals of h_{ij}^{TT} . The full solutions are certainly not spatially homogeneous. However, every spatial inhomogeneity means that a pair of transverse-traceless free fields has failed to contribute an infrared logarithm, which is spatially homogeneous. Hence the leading logarithm result should take the form (48-49) we have assumed.
- The g_{00} constraint equation of motion would ordinarily determine a linear combination of $h^L(t, \vec{x})$ and $h(t, \vec{x})$, so it is natural to use it to infer the effective scale factor $A(t)$. Taking the expectation value at leading logarithm order ensures consistency of the method.

It will be seen that the Effective Scale Factor Approximation provides the possibility for a self-consistent test of the “null hypothesis” that infrared logarithms make no significant corrections to inflationary quantum gravity [19]. For if the null hypothesis were correct then either the effective scale factor should remain at its de Sitter value — and the graviton should remain free — or else there are corrections but they drop out of a gauge invariant measure of back-reaction. In sections 4 and 5 we show that neither supposition is correct.

Even though the long wavelength limit of a graviton is gauge equivalent to zero, it will be seen that *each term in the graviton expansion of equation (45) contains two differentiated gravitons*. So any effect is sourced by the nonzero values of these derivatives and would indeed vanish for a purely constant graviton. However, recall that the expectation values of differentiated free

fields cannot be infrared truncated, so one has instead:

$$\langle \Omega | \dot{\chi}_{ij}(t, \vec{x}) \dot{\chi}_{kl}(t, \vec{x}) | \Omega \rangle \sim H^4 , \quad (50)$$

$$\langle \Omega | \partial_m \chi_{ij}(t, \vec{x}) \partial_n \chi_{kl}(t, \vec{x}) | \Omega \rangle \sim a^2(t) H^4 . \quad (51)$$

These are still only small corrections, but they get enhanced by infrared logarithms at higher orders and the final effect may become large at late times.

3.3 VEVs versus Stochastic Samples

Inflationary cosmologists sometimes object to taking expectation values to follow evolution of a homogeneous, mean geometry, as we do in (45). They maintain that a long period of cosmological evolution presents a sort of Schrodinger Cat Paradox in which the wave function of the universe decoheres into many different portions, which are no longer in good quantum contact with one another, and in which the geometry is not even approximately spatially homogeneous on super-horizon scales [38]. There is no question that this view is correct at some level because the vacuum expectation value of the stress-energy tensor is homogeneous and isotropic (because the state is) whereas we perceive (indeed we *are*) inhomogeneities and anisotropies. The key issues which have not been quantitatively addressed, and which bear on the validity of the present analysis, are:

1. How unreliable are expectation values? and
2. How much spatial variation should one expect?

We suspect that the reliability of expectation values depends upon the operator under study. For operators which average to zero, such as the density perturbation, the entire result arises from the decoherence effect, so one makes an enormous mistake in ignoring it. Other operators — for example, the square of a scalar field — acquire a significant homogeneous expectation value upon which spatial variations are superimposed. Any quantum fluctuation drives this sort of operator positive, so one might happen to inhabit a special region of the universe in which there is little effect for a long time, but there will sooner or later be a large effect. The expectation value of such an operator can correctly reflect the long-term trend everywhere in space, even though it misses variations from one region to another.

The example of $\varphi^2(t, \vec{x})$ is not specious because the free expectation value of this operator is one of the two ways infrared logarithms arise in perturbative computations. An even more likely candidate for the general reliability of expectation values is the gravitational interaction between the infrared gravitons which everyone agrees are produced during inflation. The self-gravitation of these particles must slow inflation, even if their distribution is not perfectly homogeneous. And this self-gravitation must grow with time as more and more infrared gravitons are ripped out of the vacuum by inflation. As discussed in section 1, the fact that the total kinetic energy of infrared gravitons grows like the cube of the universe's radius means that the entire universe rather quickly falls inside its own Schwarzschild radius. The actual gravitational potential grows like $\ln(a)$, rather than a^2 , because causality delays effects from a patch of spacetime until it passes within one's past light-cone [18]. Because gravity is a weak interaction, even at the scales of primordial inflation, the self-gravitation of inflationary gravitons cannot be significant until the past light-cone has become enormous. So a significant back-reaction from quantum gravity can only occur as the cumulative effect of very many small fluctuations within the past light-cone of the observation point. Because the past light-cones of nearby points must largely overlap, there cannot be much spatial variation within a few Hubble radii. There might well be significant variation on very large scales, but even this should not change the long-term slowing trend everywhere. So we believe that quantifying this trend through the use of expectation values does not represent a serious error.

We emphasize that the reliability of expectation values is a quantitative issue rather than a qualitative one. We do *not* maintain that the actual universe is exactly homogeneous and isotropic, as its quantum average is. Rather we argue that using expectation values is unlikely to produce significant errors for the specific back-reaction effect we seek to study. Far from ignoring it, the inherently stochastic spatial variation of back-reaction plays an essential role in our model of inflation: *it is how one can get scalar density perturbations of about the correct strength without the need for a fundamental scalar.*

To understand what we have in mind, suppose that the leading infrared logarithms grow to become nonperturbatively strong and that they sum up to produce a graceful exit to inflation. As noted above, there may be large spatial variations on vastly super-horizon scales, but the general trend should be correctly described by the average geometry. Because the slowing effect

derives from an enormous number of small fluctuations generated over the past light-cone of a very large ($\sim 10^{12}$) number of e-foldings, there can be only small spatial variation, even over regions as large as the currently observable universe. (Because the ~ 60 e-foldings after the end of primordial inflation is very much smaller than the $\sim 10^{12}$ we envisage to end inflation.) But there is *some* spatial variation; what is its likely strength? Because the leading infrared logarithms are spatially homogeneous and isotropic, perturbations around them must be subdominant. In other words, inhomogeneities must be suppressed by at least one power of GH^2 . Because the leading logarithms are assumed to give an order one effect (stopping inflation) the first subleading terms should induce a spectrum of density perturbations of order GH^2 . *This is what one wants!* The power spectra for scalar and tensor perturbations of wave number k can be approximately expressed in terms of the values of the Hubble parameter and deceleration parameter at the time t_k when that mode experienced first horizon crossing [37]

$$\mathcal{P}_S(k) \sim \frac{GH^2(t_k)}{1 + q(t_k)} \quad , \quad \mathcal{P}_T(k) \sim GH^2(t_k) . \quad (52)$$

There is no way to recognize a factor such as $1/[1 + q(t_k)]$, and this level of analysis, but the factors of $GH^2(t_k)$ come out exactly right.

4 Computing the Effective Scale Factor

The purpose of this section is to solve equation (45) for the effective scale factor $A(t)$. Because differentiated fields must be treated differently than undifferentiated ones, we begin by isolating the derivatives. We then explain how the various indices can be factored out to reduce any 2-point correlator to one involving two massless, minimally coupled scalars. Next we give a brief description of how the equation can be formulated for a nonperturbative, numerical evolution. We close by working out the 1- and 2-loop corrections.

4.1 Isolating the Derivatives

In synchronous gauge ($g_{00} = -1$, $g_{0i} = 0$) the purely temporal component of the Einstein tensor $G_{\mu\nu}$ takes the form:

$$G_{00} \equiv R_{00} - \frac{1}{2}g_{00}R = \frac{1}{8}g^{ij}g^{k\ell}(\dot{g}_{ij}\dot{g}_{k\ell} - \dot{g}_{ik}\dot{g}_{j\ell}) + \frac{1}{2}^{(D-1)}R \quad , \quad (53)$$

where $^{(D-1)}R$ is the Ricci scalar constructed from the $(D-1)$ -dimensional spatial metric $g_{ij}(t, \vec{x})$. We now extract the effective scale factor:

$$g_{ij}(t, \vec{x}) \equiv A^2(t) \bar{g}_{ij}(t, \vec{x}) , \quad (54)$$

so that:

$$\begin{aligned} G_{00} = & \frac{1}{2}(D-1)(D-2)\frac{\dot{A}^2}{A^2} + \frac{1}{2}(D-2)\frac{\dot{A}}{A}\bar{g}^{ij}\dot{\bar{g}}_{ij} \\ & + \frac{1}{8}\bar{g}^{ij}\bar{g}^{kl}(\dot{\bar{g}}_{ij}\dot{\bar{g}}_{kl} - \dot{\bar{g}}_{ik}\dot{\bar{g}}_{jl}) + \frac{1}{2A^2}^{(D-1)}\bar{R} , \end{aligned} \quad (55)$$

where the spatial Ricci scalar is:

$$^{(D-1)}\bar{R} \equiv \bar{g}^{ij}\bar{g}^{kl}\left[\bar{\Gamma}_{\ell ji,k} - \bar{\Gamma}_{\ell ki,j}\right] + \bar{g}^{ij}\bar{g}^{kl}\bar{g}^{mn}\left[\bar{\Gamma}_{mjk}\bar{\Gamma}_{nli} - \bar{\Gamma}_{mij}\bar{\Gamma}_{nkl}\right] , \quad (56)$$

$$\begin{aligned} & = \bar{g}^{ij}\bar{g}^{kl}\left[\bar{g}_{ik,j\ell} - \bar{g}_{ij,k\ell}\right] \\ & + \bar{g}^{ij}\bar{g}^{kl}\bar{g}^{mn}\left[\frac{1}{4}\bar{g}_{ik,m}\bar{g}_{j\ell,n} - \frac{1}{4}\bar{g}_{ij,m}\bar{g}_{k\ell,n} - \bar{g}_{ik,\ell}\bar{g}_{jm,n} + \bar{g}_{ik,\ell}\bar{g}_{mn,i}\right] . \end{aligned} \quad (57)$$

At this point we are ready to substitute the Effective Scale Factor Approximation form (43) for the metric. The fact that \bar{g}_{ij} is the exponential of a traceless field means that:

- (i) we can drop all derivatives of the form $\bar{g}^{ij}\partial\bar{g}_{ij}$,
- (ii) we can convert the second derivative terms to a total derivative – whose expectation value must vanish – plus products of first derivatives:

$$\begin{aligned} \bar{g}^{ij}\bar{g}^{kl}\left[\bar{g}_{ik,j\ell} - \bar{g}_{ij,k\ell}\right] = \\ - \bar{g}^{ij}_{,ij} + \bar{g}^{ij}\bar{g}^{kl}\bar{g}^{mn}\left[\bar{g}_{ik,\ell}\bar{g}_{jm,n} + \bar{g}_{ik,m}\bar{g}_{n\ell,j} - \bar{g}_{ik,m}\bar{g}_{j\ell,n}\right] . \end{aligned} \quad (58)$$

As a result, the equation for $A(t)$ becomes:

$$0 = \langle\langle \Omega | G_{00} | \Omega \rangle\rangle - \frac{1}{2}(D-2)\left(\Lambda + \delta\Lambda_1\kappa^2 H^4\right) , \quad (59)$$

$$\begin{aligned} & = \frac{1}{2}(D-2)\left[(D-1)\frac{\dot{A}^2}{A^2} - \Lambda - \delta\Lambda_1\kappa^2 H^4\right] - \frac{1}{8}\langle\langle \Omega | \bar{g}^{ij}\bar{g}^{kl}\dot{\bar{g}}_{jk}\dot{\bar{g}}_{\ell i} | \Omega \rangle\rangle \\ & + \frac{1}{2A^2}\langle\langle \Omega | \bar{g}^{ij}\bar{g}^{kl}\bar{g}^{mn}\left(-\frac{3}{4}\bar{g}_{ik,m}\bar{g}_{j\ell,n} + \bar{g}_{ik,m}\bar{g}_{n\ell,j}\right) | \Omega \rangle\rangle . \end{aligned} \quad (60)$$

It remains to act the derivatives on specific $H_{ij}[A](t, \vec{x})$ fields and isolate the expectation values of the differentiated fields. For this purpose, it is

useful to recall a standard formula for the derivative of the exponential of a matrix M_{ij} :

$$\partial e^M = e^M \left\{ \partial M + \frac{1}{2!} [\partial M, M] + \frac{1}{3!} [[\partial M, M], M] + \dots \right\} . \quad (61)$$

We can define the term on the right hand side as the differentiated matrix contracted into a 4-index object $B[M]_{ijkl}$:

$$\partial(e^M)_{ij} \equiv (e^M)_{ik} \times B[M]_{kjpq} \times \partial M_{pq} , \quad (62)$$

which implies:

$$\begin{aligned} B[M]_{ijkl} &= \delta_{ik} \delta_{lj} + \frac{1}{2!} [\delta_{ik} M_{lj} - M_{ik} \delta_{lj}] \\ &\quad + \frac{1}{3!} [\delta_{ik} (M^2)_{lj} - 2M_{ik} M_{lj} + (M^2)_{ik} \delta_{lj}] + \dots \end{aligned} \quad (63)$$

The 4-index symbol $B[\kappa H]_{ijkl}$ is particularly useful for expressing derivatives of $\bar{g}_{ij} = \exp[\kappa H]_{ij}$ because the differentiated metrics in (60) are contracted into inverse metrics:

$$\bar{g}^{ij} \bar{g}^{kl} \dot{\bar{g}}_{jk} \dot{\bar{g}}_{li} = B[\kappa H]_{ijkl} B[\kappa H]_{jipq} \times \kappa^2 \dot{H}_{kl} \dot{H}_{pq} , \quad (64)$$

$$\bar{g}^{ij} \bar{g}^{kl} \bar{g}^{mn} \bar{g}_{ik,m} \bar{g}_{jl,n} = B[\kappa H]_{ijkl} B[\kappa H]_{jipq} \bar{g}^{mn} \times \kappa^2 H_{kl,m} H_{pq,n} , \quad (65)$$

$$\bar{g}^{ij} \bar{g}^{kl} \bar{g}^{mn} \bar{g}_{ik,m} \bar{g}_{nl,j} = B[\kappa H]_{nikl} B[\kappa H]_{mjipq} \bar{g}^{ij} \times \kappa^2 H_{kl,m} H_{pq,n} . \quad (66)$$

It follows that the equation for the effective scale factor can be written as:

$$\begin{aligned} \frac{1}{2}(D-2) \left[(D-1) \frac{\dot{A}^2}{A^2} - \Lambda - \delta\Lambda_1 \kappa^2 H^4 \right] &= \\ \frac{1}{8} \langle\langle \Omega | B[\kappa H]_{ijkl} B[\kappa H]_{jipq} | \Omega \rangle\rangle \times \langle \Omega | \kappa^2 \dot{H}_{kl} \dot{H}_{pq} | \Omega \rangle & \\ + \langle\langle \Omega | \frac{3}{8} B[\kappa H]_{ijkl} B[\kappa H]_{jipq} \bar{g}^{mn} - \frac{1}{2} B[\kappa H]_{nikl} B[\kappa H]_{mjipq} \bar{g}^{ij} | \Omega \rangle\rangle & \\ \times \frac{1}{A^2} \langle \Omega | \kappa^2 H_{kl,m} H_{pq,n} | \Omega \rangle . & \end{aligned} \quad (67)$$

4.2 Factoring Out the Indices

Since the mode functions $u[A](t, k)$ depend upon \vec{k} only through its magnitude, we can use rotational invariance to reduce expectation values of

$H_{ij}[A](t, \vec{x})$ to those of a fictitious scalar mode sum:

$$\varphi(t, \vec{x}) \equiv \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \theta(k-H) \left\{ u[A](t, k) e^{i\vec{k}\cdot\vec{x}} \alpha(\vec{k}) + (cc) \right\} . \quad (68)$$

This is achieved by writing the expectation value as an integral of the form:

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} f(k) \hat{k}_i \hat{k}_j \cdots , \quad (69)$$

and then extracting the indices through the familiar replacements:

$$\hat{k}_i \hat{k}_j \longmapsto \frac{1}{D-1} \delta_{ij} , \quad (70)$$

$$\hat{k}_i \hat{k}_j \hat{k}_k \hat{k}_\ell \longmapsto \frac{1}{(D+1)(D-1)} \left[\delta_{ij} \delta_{k\ell} + \delta_{ik} \delta_{j\ell} + \delta_{i\ell} \delta_{jk} \right] , \quad (71)$$

$$\hat{k}_i \hat{k}_j \hat{k}_k \hat{k}_\ell \hat{k}_m \hat{k}_n \longmapsto \frac{1}{(D+3)(D+1)(D-1)} \left[\delta_{ij} \delta_{k\ell} \delta_{mn} + \cdots \right] . \quad (72)$$

From the mode sum (44) and definitions (10-11) we can evaluate the coincident 2-point functions of interest:

(i) The undifferentiated 2-point coincident function,

$$\begin{aligned} & \langle \Omega | H_{ij}[A](t, \vec{x}) H_{k\ell}[A](t, \vec{x}) | \Omega \rangle \\ &= 2 \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \|u[A](t, k)\|^2 \left[\Pi_{i(k)\ell j} - \frac{\Pi_{ij}\Pi_{k\ell}}{D-2} \right] , \end{aligned} \quad (73)$$

$$= \frac{2D(D-3)}{(D+1)(D-2)} \left[\delta_{i(k)\ell j} - \frac{\delta_{ij}\delta_{k\ell}}{D-1} \right] \times \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \|u[A](t, k)\|^2 , \quad (74)$$

$$\equiv \frac{2D(D-3)}{(D+1)(D-2)} \left[\delta_{i(k)\ell j} - \frac{\delta_{ij}\delta_{k\ell}}{D-1} \right] \times \langle \varphi^2 \rangle . \quad (75)$$

(ii) The “temporal derivatives” coincident 2-point function,

$$\begin{aligned} & \langle \Omega | \dot{H}_{ij}[A](t, \vec{x}) \dot{H}_{k\ell}[A](t, \vec{x}) | \Omega \rangle \\ &= \frac{2D(D-3)}{(D+1)(D-2)} \left[\delta_{i(k)\ell j} - \frac{\delta_{ij}\delta_{k\ell}}{D-1} \right] \times \langle \dot{\varphi}^2 \rangle . \end{aligned} \quad (76)$$

(iii) The “spatial derivatives” coincident 2-point function,

$$\begin{aligned}
& \left\langle \Omega \left| \partial_m H_{ij}[A](t, \vec{x}) \partial_n H_{k\ell}[A](t, \vec{x}) \right| \Omega \right\rangle \\
&= \frac{2D}{(D+3)(D+1)(D-1)(D-2)} \left[-(D+1)\delta_{ij}\delta_{k\ell}\delta_{mn} + 4\delta_{ij}\delta_{k(m}\delta_{n)\ell} \right. \\
& \quad \left. + 4\delta_{k\ell}\delta_{i(m}\delta_{n)j} + (D^2-5)\delta_{i(k}\delta_{\ell)j}\delta_{mn} - 4(D-1)\delta_{i(k}\delta_{\ell)(m}\delta_{n)(j)} \right] \times \left\langle \|\vec{\nabla}\varphi\|^2 \right\rangle .
\end{aligned} \tag{77}$$

4.3 Nonperturbative Formulation

The fictitious scalar expectation values in expressions (75-77) can be written as one dimensional integrals involving the mode functions $u[A](t, k)$ for general $A(t)$:

$$\left\langle \varphi^2 \right\rangle = \frac{1}{2^{D-2} \pi^{(\frac{D-1}{2})} \Gamma(\frac{D-1}{2})} \int_H^\infty dk k^{D-2} \left\| u[A](t, k) \right\|^2 , \tag{78}$$

$$\left\langle \dot{\varphi}^2 \right\rangle = \frac{1}{2^{D-2} \pi^{(\frac{D-1}{2})} \Gamma(\frac{D-1}{2})} \int_H^\infty dk k^{D-2} \left\| \dot{u}[A](t, k) \right\|^2 , \tag{79}$$

$$\left\langle \|\vec{\nabla}\varphi\|^2 \right\rangle = \frac{1}{2^{D-2} \pi^{(\frac{D-1}{2})} \Gamma(\frac{D-1}{2})} \int_H^\infty dk k^D \left\| u[A](t, k) \right\|^2 . \tag{80}$$

Some work remains to be done converting the expression for $u[A](t, k)$ obtained in [37, 39] to a form for which these integrals are useful, but this seems straightforward. In the remainder of this section we explain how (78) can be used to compute leading log expectation values of nonlinear functions of the undifferentiated free field $H_{ij}[A](t, \vec{x})$ such as those in the nonperturbative evolution equation (67).

Let us begin with the expectation value of some analytic function $F(x)$ of the fictitious scalar $\varphi(t, \vec{x})$. Because the function is analytic we have:

$$F(\varphi) \equiv \sum_{n=0}^{\infty} \frac{F^n(0)\varphi^n}{n!} . \tag{81}$$

Because the fictitious scalar is free we have:

$$\left\langle \varphi^{2N-1} \right\rangle = 0 , \quad \left\langle \varphi^{2N} \right\rangle = (2N-1)!! \times \left\langle \varphi^2 \right\rangle^N . \tag{82}$$

Thus, the expectation value of $F(\varphi)$ is:

$$\left\langle F(\varphi) \right\rangle = \sum_{n=0}^{\infty} \frac{F^{2n}(0)}{2^n n!} \times \left\langle \varphi^2 \right\rangle^n . \tag{83}$$

In view of the identity:

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dy e^{-y^2} y^{2n} = \frac{(2n-1)!!}{2^n} , \quad (84)$$

we reach the final form:

$$\langle F(\varphi) \rangle = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dy e^{-y^2} F\left(y \sqrt{2\langle \varphi^2 \rangle}\right) . \quad (85)$$

We seek now the expectation value of an analytic function of the matrix of free fields $H_{ij}[A](t, \vec{x})$. Since they are free, any such expectation value will break up into sums of products of expectation values of pairs of these fields, possibly with different indices. Expression (75) allows us to reduce the expectation value of any H_{ij} pair to a tensor constant times $\langle \varphi^2 \rangle$. There are many integral representations analogous to (85) for correctly representing the combinatoric factors. Rather than integrating over a traceless dummy matrix, we choose the simpler option of integrating over a symmetric dummy variable Q_{ij} and then enforcing tracelessness directly:

$$\begin{aligned} \langle F(H_{ij}) \rangle = & \\ \frac{1}{\pi^{\frac{D^2-D}{4}}} \int [dQ_{ij}] e^{-Q_{ij}Q_{ij}} F\left(\left[Q_{ij} - \frac{\delta_{ij} Q_{kk}}{D-1}\right] \sqrt{\frac{4D(D-3)}{(D+1)(D-2)} \langle \varphi^2 \rangle}\right) . & (86) \end{aligned}$$

There is no need to continue working in D dimensions since we are only interested in the ultraviolet finite, leading logarithm contributions from such expectation values. We can therefore take $D = 4$:

$$\langle\langle F(H_{ij}) \rangle\rangle = \frac{1}{\pi^3} \int [dQ_{ij}] e^{-Q_{ij}Q_{ij}} F\left(\left[Q_{ij} - \frac{1}{3}\delta_{ij} Q_{kk}\right] \sqrt{\frac{8}{5} \langle\langle \varphi^2 \rangle\rangle}\right) . \quad (87)$$

It is also very convenient to transform Q to a diagonal form \mathcal{D} via a 3-dimensional rotation \mathcal{R} :

$$\mathcal{R}_{ij} \equiv (\delta_{ij} - \hat{\theta}_i \hat{\theta}_j) \cos(\theta) + \epsilon_{ijk} \hat{\theta}_k \sin(\theta) + \hat{\theta}_i \hat{\theta}_j , \quad (88)$$

$$Q = \mathcal{R} \mathcal{D} \mathcal{R}^T , \quad Q_{ij} = \mathcal{R}_{ik} \mathcal{D}_{kl} \mathcal{R}_{jl} , \quad (89)$$

This is advantageous because the rotation resides only on free indices in complicated functions such as the inverse metric and the 4-index object $B[M]_{ijkl}$

of expression (63):

$$\exp\left[-\kappa Q + \frac{1}{3}\text{Tr}(\kappa Q) I\right]_{ij} = \mathcal{R}_{im}\mathcal{R}_{jn} \times \exp\left[-\kappa \mathcal{D} + \frac{1}{3}\text{Tr}(\kappa \mathcal{D}) I\right]_{mn} , \quad (90)$$

$$B\left[\kappa Q - \frac{1}{3}\text{Tr}(\kappa Q) I\right]_{ijk\ell} = \mathcal{R}_{im}\mathcal{R}_{jn}\mathcal{R}_{kp}\mathcal{R}_{\ell q} \times B\left[\kappa \mathcal{D} - \frac{1}{3}\text{Tr}(\kappa \mathcal{D}) I\right]_{mnpq} \quad (91)$$

and drops entirely out of the trace of Q_{ij} in the exponential:

$$Q_{ij} Q_{ij} = \mathcal{D}_{11}^2 + \mathcal{D}_{22}^2 + \mathcal{D}_{33}^2 \equiv \mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2 . \quad (92)$$

The measure factor in (87) takes the form:

$$\int d^6 Q = \int d^3 \mathcal{R} \int d^3 \mathcal{D} J , \quad (93)$$

where J is the Jacobian determinant of the 6×6 matrix of derivatives of the six Q_{ij} 's with respect to the six variables $(\theta^i, \mathcal{D}_j)$. Because J is independent of θ^i we can compute it using the small angle approximation:

$$\mathcal{R}_{ij} = \delta_{ij} + \epsilon_{ijk}\theta_k + O(\theta^2) . \quad (94)$$

Then the transformation (89) gives:

$$\begin{aligned} Q_{ii} &= \mathcal{D}_i + O(\theta^2) , \\ Q_{12} = Q_{21} &= -(\mathcal{D}_1 - \mathcal{D}_2) \theta_3 + O(\theta^3) , \\ Q_{23} = Q_{32} &= -(\mathcal{D}_2 - \mathcal{D}_3) \theta_1 + O(\theta^3) , \\ Q_{31} = Q_{13} &= -(\mathcal{D}_3 - \mathcal{D}_1) \theta_2 + O(\theta^3) . \end{aligned} \quad (95)$$

The Jacobian determinant is therefore:

$$J = |(\mathcal{D}_1 - \mathcal{D}_2)(\mathcal{D}_2 - \mathcal{D}_3)(\mathcal{D}_3 - \mathcal{D}_1)| . \quad (96)$$

Integrating out the angular variables gives a constant N :

$$\begin{aligned} \int [dQ_{ij}] e^{-Q_{ij}Q_{ij}} &= \\ N \int d^3 \mathcal{D} |(\mathcal{D}_1 - \mathcal{D}_2)(\mathcal{D}_2 - \mathcal{D}_3)(\mathcal{D}_3 - \mathcal{D}_1)| e^{-(\mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2)} . \end{aligned} \quad (97)$$

We can find N by transforming to average and relative variables:

$$\bar{\mathcal{D}} \equiv (\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3) \quad ; \quad \Delta_1 \equiv (\mathcal{D}_1 - \mathcal{D}_2) \quad , \quad \Delta_2 \equiv (\mathcal{D}_2 - \mathcal{D}_3) \quad , \quad (98)$$

so that:

$$\begin{aligned}
& \int d^3\mathcal{D} \left| (\mathcal{D}_1 - \mathcal{D}_2)(\mathcal{D}_2 - \mathcal{D}_3)(\mathcal{D}_3 - \mathcal{D}_1) \right| e^{-(\mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2)} \\
&= \int_{-\infty}^{\infty} d\bar{\mathcal{D}} e^{-3\bar{\mathcal{D}}^2} \int_{-\infty}^{\infty} d\Delta_1 \int_{-\infty}^{\infty} d\Delta_2 \left| \Delta_1 \Delta_2 (\Delta_1 + \Delta_2) \right| e^{-\frac{2}{3}(\Delta_1^2 + \Delta_1 \Delta_2 + \Delta_2^2)} \\
&= \sqrt{\frac{\pi}{3}} \int_{-\infty}^{\infty} d\Delta_1 \int_{-\infty}^{\infty} d\Delta_2 \left| \Delta_1 \Delta_2 (\Delta_1 + \Delta_2) \right| e^{-\frac{2}{3}(\Delta_1^2 + \Delta_1 \Delta_2 + \Delta_2^2)} . \tag{99}
\end{aligned}$$

We then change to sum and difference variables:

$$\alpha \equiv \Delta_1 + \Delta_2 \quad , \quad \beta \equiv \Delta_1 - \Delta_2 \quad , \tag{100}$$

and evaluate the trivial Gaussian integrals:

$$\begin{aligned}
& \int d^3\mathcal{D} \left| (\mathcal{D}_1 - \mathcal{D}_2)(\mathcal{D}_2 - \mathcal{D}_3)(\mathcal{D}_3 - \mathcal{D}_1) \right| e^{-(\mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2)} \\
&= \sqrt{\frac{\pi}{3}} \frac{1}{2} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \frac{1}{4} \left| (\alpha^2 - \beta^2)\alpha \right| e^{-\frac{1}{2}\alpha^2 - \frac{1}{6}\beta^2} = \frac{3\pi}{\sqrt{8}} . \tag{101}
\end{aligned}$$

Equation (101) implies that the constant N in (97) is:

$$N = \frac{\sqrt{8}}{3\pi} . \tag{102}$$

4.4 The First Loop

It is a simple matter to evaluate the right hand side of (67) at one loop – that is, at order κ^2 . Only the zeroth order parts of the inverse metric and the 4-index symbol (63) can contribute:

$$\bar{g}^{mn} \longrightarrow \delta^{mn} \quad , \quad B_{ijkl}[\kappa H] \longrightarrow \delta_{ik}\delta_{lj} . \tag{103}$$

Taking account of the fact that $H_{ij}[A](t, \vec{x})$ is transverse and using expressions (76-77) gives:

$$\begin{aligned}
& \frac{1}{2}(D-2) \left[(D-1) \frac{\dot{A}^2}{A^2} - \Lambda - \delta\Lambda_1 \kappa^2 H^4 \right] \\
&= \frac{1}{8} \left\langle \Omega \left| \kappa^2 \dot{H}_{ij} \dot{H}_{ij} \right| \Omega \right\rangle + \frac{3}{8A^2} \left\langle \Omega \left| \kappa^2 H_{ij,m} H_{ij,m} \right| \Omega \right\rangle + O(\kappa^4) , \tag{104}
\end{aligned}$$

$$= \frac{D(D-3)}{8} \left\{ \left\langle \kappa^2 \dot{\varphi}^2 \right\rangle + \frac{3}{A^2} \left\langle \kappa^2 \|\vec{\nabla}\varphi\|^2 \right\rangle \right\} + O(\kappa^4) . \tag{105}$$

Expectation values of the fictitious scalar depend upon the effective scale factor $A(t)$, which is itself a series in κ^2 . However, at this order we require only the well known de Sitter limit [6]:

$$\left\langle \kappa^2 \partial_\mu \varphi \partial_\nu \varphi \right\rangle_{A=a} = -\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D)}{\Gamma(\frac{D}{2}+1)} \frac{1}{2} \kappa^2 H^2 g_{\mu\nu}^{\text{dS}} , \quad (106)$$

where:

$$g_{\mu\nu}^{\text{dS}} dx^\mu dx^\nu \equiv -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} . \quad (107)$$

Substituting in (105) gives:

$$\begin{aligned} \frac{D-2}{2} \left[(D-1) \frac{\dot{A}^2}{A^2} - \Lambda - \delta\Lambda_1 \kappa^2 H^4 \right] \\ = \frac{\kappa^2 H^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D+1)}{\Gamma(\frac{D}{2}+1)} \frac{(4-3D)(D-3)}{16} + O(\kappa^4) . \end{aligned} \quad (108)$$

Since all factors in the above equation are finite, we can also set $D = 4$:

$$3 \frac{\dot{A}^2}{A^2} - 3H^2 - \delta\Lambda_1 \kappa^2 H^4 = -\frac{3\kappa^2 H^4}{8\pi^2} + O(\kappa^4) . \quad (109)$$

Furthermore, we express (109) more simply as an equation for the square of \dot{A}/A , and we can more fully anticipate the form of the next correction:

$$\frac{\dot{A}^2}{A^2} = H^2 \left\{ 1 + \frac{1}{3} \delta\Lambda_1 \kappa^2 H^2 - \frac{\kappa^2 H^2}{8\pi^2} + O(\kappa^4 H^4 \ln(a)) \right\} . \quad (110)$$

At this point we should comment on various other 1-loop computations. Had we chosen $\delta\Lambda_1$ to null the 1-loop correction to $(\dot{A}/A)^2$ the result would be:

$$(\delta\Lambda_1)_{\text{synchronous}} = +\frac{3}{8\pi^2} . \quad (111)$$

A previous computation in a covariant gauge found [40]:

$$(\delta\Lambda_1)_{\text{covariant}} = -\frac{3}{2\pi^2} . \quad (112)$$

Finelli, Marozzi, Venturi and Vacca worked in synchronous gauge but they used an adiabatic regularization to get [41]:

$$(\delta\Lambda_1)_{\text{FMVV}} = -\frac{361}{1920\pi^2} . \quad (113)$$

These differences reflect the well-known fact that counterterms can depend upon the choice of gauge and upon the regularization technique. The physically significant fact is the universal agreement that any 1-loop shift in $(\dot{A}/A)^2$ can be absorbed into $\delta\Lambda_1$.

One can also estimate the result from just infrared gravitons. A very early computation by Ford got [42]:

$$\left(\delta\Lambda_1\right)_{\text{Ford}} = +\frac{1}{\pi^2} . \quad (114)$$

By instead modeling each graviton polarization as a massless, minimally coupled scalar and restricting to the infrared, we find [18]:

$$\left(\delta\Lambda_1\right)_{\text{infrared}} = -\frac{1}{16\pi^2} . \quad (115)$$

The reason (111) and (115) differ in sign highlights an important aspect of the leading logarithm approximation. Expression (105) reveals that the 1-loop effect consists of a sum of the expectation values of the squares of two operators. If one makes an infrared truncation of the resulting mode sums this must give a positive effect, which could only be nulled by a negative value of $\delta\Lambda_1$. However, that is not the correct way to work at leading logarithm order. It is *always* the full quantum field theory which defines results; the restriction to infrared modes is only justified when extracting an infrared logarithm. The differentiated operators that appear in expression (105) cannot produce infrared logarithms. What they give instead is constants, and the values of these constants derive from the ultraviolet as well as the infrared. The correct procedure is to compute the expectation value from the full mode sum, with the regularization in place. When this is done one can see from expression (106) that the expectation value of $\|\vec{\nabla}\varphi\|^2$ is actually *negative*, and large enough that it predominates over the positive expectation value of $\dot{\varphi}^2$. Of course there is no mystery about the expectation value of a square giving a negative result; the automatic subtraction of dimensional regularization has subsumed the positive, power law divergence – which does not, in any case, contribute as vacuum energy – and left a finite, negative remainder.

Whether or not we make the choice (111) to null the 1-loop contribution to \dot{A}/A , it is still constant. This allows us to evaluate the derivatives of expression (105) to higher order using the de Sitter result (106) with H

replaced by \dot{A}/A :

$$\langle \kappa^2 \dot{\varphi}^2 \rangle = + \frac{3\kappa^2}{32\pi^2} \frac{\dot{A}^4}{A^4} + O(\kappa^6) , \quad (116)$$

$$\frac{3}{A^2} \langle \kappa^2 \|\vec{\nabla} \varphi\|^2 \rangle = - \frac{27\kappa^2}{32\pi^2} \frac{\dot{A}^4}{A^4} + O(\kappa^6) . \quad (117)$$

Since the 1-loop correction to \dot{A}/A is constant we can actually ignore it compared to the leading logarithm terms that derive from the undifferentiated factors of $\kappa H_{ij}[A](t, \vec{x})$ in the inverse metric and in the 4-index symbol $B[\kappa H]_{ijkl}$:

$$\langle\langle \kappa^2 \dot{\varphi}^2 \rangle\rangle = + \frac{3\kappa^2 H^4}{32\pi^2} + O(\kappa^6) , \quad (118)$$

$$\frac{3}{A^2} \langle\langle \kappa^2 \|\vec{\nabla} \varphi\|^2 \rangle\rangle = - \frac{27\kappa^2 H^4}{32\pi^2} + O(\kappa^6) . \quad (119)$$

4.5 The Second Loop

We are now ready to evaluate expression (67) at 2-loop order. A useful identity is the expansion of two contracted 4-index symbols:

$$\begin{aligned} & \langle\langle \Omega | B[\kappa H]_{ijkl} B[\kappa H]_{jipq} | \Omega \rangle\rangle \\ &= \delta_{qk} \delta_{\ell p} + \frac{\kappa^2}{12} \langle\langle \Omega | \delta_{qk} (H^2)_{\ell p} - 2H_{qk} H_{\ell p} + (H^2)_{qk} \delta_{\ell p} | \Omega \rangle\rangle + O(\kappa^4) , \end{aligned} \quad (120)$$

$$= \delta_{qk} \delta_{\ell p} + \frac{1}{15} [4\delta_{qk} \delta_{\ell p} - \delta_{q\ell} \delta_{kp} - \delta_{qp} \delta_{k\ell}] \langle\langle \kappa^2 \varphi^2 \rangle\rangle + O(\kappa^4) . \quad (121)$$

Contracting (121) into (76) and then making use of (118) gives the first of the three expansions we require:

$$\begin{aligned} & \frac{1}{8} \langle\langle \Omega | B[\kappa H]_{ijkl} B[\kappa H]_{jipq} | \Omega \rangle\rangle \times \langle \Omega | \kappa^2 \dot{H}_{k\ell} \dot{H}_{pq} | \Omega \rangle \\ &= \left\{ \frac{1}{2} + \frac{1}{10} \langle\langle \kappa^2 \varphi^2 \rangle\rangle + O(\kappa^4) \right\} \times \langle \kappa^2 \dot{\varphi}^2 \rangle , \end{aligned} \quad (122)$$

$$= \left\{ \frac{1}{2} + \frac{1}{10} \times \frac{\kappa^2 H^2}{4\pi^2} \ln(a) + O(\kappa^4 H^4 \ln^2(a)) \right\} \times \frac{3\kappa^2 H^4}{32\pi^2} . \quad (123)$$

It is by now clear that we are doing an expansion in powers of a dimensionless time dependent parameter which may as well be named:

$$x(t) \equiv \frac{\kappa^2 H^2}{4\pi^2} \ln(a) . \quad (124)$$

It is also clear that every term in our expansion will involve one factor of $\dot{x}H$. (Higher derivatives can and do appear in subdominant logarithm corrections, but they cannot occur at leading order because every time derivative eliminates a factor of $t = \ln(a)/H$. This is another way of seeing that only a renormalization of the cosmological constant is required at leading logarithm order.) With this notation the first of our expansions (123) assumes the simple form:

$$\begin{aligned} \frac{1}{8} \langle\langle \Omega | B[\kappa H]_{ijkl} B[\kappa H]_{jipq} | \Omega \rangle\rangle &\times \langle \Omega | \kappa^2 \dot{H}_{k\ell} \dot{H}_{pq} | \Omega \rangle \\ &= H\dot{x} \left\{ \frac{3}{16} + \frac{3}{80} x + O(x^2) \right\} . \end{aligned} \quad (125)$$

The inclusion of an inverse metric in relation (121) adds only a single extra term at the order we are working:

$$\begin{aligned} &\langle\langle \Omega | B[\kappa H]_{ijkl} B[\kappa H]_{jipq} \bar{g}^{mn} | \Omega \rangle\rangle \\ &= \delta_{qk} \delta_{\ell p} \delta_{mn} + \frac{\kappa^2}{2} \langle\langle \Omega | \delta_{qk} \delta_{\ell p} (H^2)_{mn} | \Omega \rangle\rangle \\ &\quad + \frac{\kappa^2}{12} \delta_{mn} \langle\langle \Omega | \delta_{qk} (H^2)_{\ell p} - 2H_{qk} H_{\ell p} + (H^2)_{qk} \delta_{\ell p} | \Omega \rangle\rangle + O(\kappa^4) , \end{aligned} \quad (126)$$

$$= \delta_{qk} \delta_{\ell p} \delta_{mn} + \frac{1}{15} [14\delta_{qk} \delta_{\ell p} - \delta_{q\ell} \delta_{kp} - \delta_{qp} \delta_{k\ell}] \delta_{mn} \langle\langle \kappa^2 \varphi^2 \rangle\rangle + O(\kappa^4) , \quad (127)$$

$$= \delta_{qk} \delta_{\ell p} \delta_{mn} + \frac{1}{15} [14\delta_{qk} \delta_{\ell p} - \delta_{q\ell} \delta_{kp} - \delta_{qp} \delta_{k\ell}] \delta_{mn} x + O(x^2) . \quad (128)$$

The overall factor of δ_{mn} in (128) gives a simple result when contracted into the derivative term (77):

$$\begin{aligned} &\frac{1}{A^2} \langle \Omega | \kappa^2 \partial_m H_{k\ell} \partial_m H_{pq} | \Omega \rangle \\ &= \frac{2D(D-3)}{(D+1)(D-2)} \left[\delta_{k(p} \delta_{q)\ell} - \frac{\delta_{k\ell} \delta_{pq}}{D-1} \right] \times \frac{1}{A^2} \langle \kappa^2 \|\vec{\nabla} \varphi\|^2 \rangle . \end{aligned} \quad (129)$$

We can take the $D = 4$ limit of the above equation because it is finite:

$$\begin{aligned} &\frac{1}{A^2} \langle \Omega | \kappa^2 \partial_m H_{k\ell} \partial_m H_{pq} | \Omega \rangle \\ &\longrightarrow -\frac{9}{10} H\dot{x} \left[\delta_{k(p} \delta_{q)\ell} - \frac{1}{3} \delta_{k\ell} \delta_{pq} \right] + O(H\dot{x}x) . \end{aligned} \quad (130)$$

Multiplying the two factors (128, 130) gives the second of the three expansions we require for equation (67):

$$\begin{aligned} \frac{3}{8} \langle\langle \Omega | B[\kappa H]_{ijkl} B[\kappa H]_{jipq} \bar{g}^{mn} | \Omega \rangle\rangle &\times \frac{1}{A^2} \langle \Omega | \kappa^2 \partial_m H_{k\ell} \partial_n H_{pq} | \Omega \rangle \\ &= H\dot{x} \left\{ -\frac{27}{16} - \frac{117}{80} x + O(x^2) \right\} . \end{aligned} \quad (131)$$

The two factors of the final term on the right hand side of (67) cannot be so usefully expanded in isolation of one another. It is better to keep them together so that transversality can be exploited. The result is:

$$\begin{aligned} -\frac{1}{2} \langle\langle \Omega | B[\kappa H]_{nik\ell} B[\kappa H]_{mjpq} \bar{g}^{ij} | \Omega \rangle\rangle &\times \frac{1}{A^2} \langle \Omega | \kappa^2 \partial_m H_{k\ell} \partial_n H_{pq} | \Omega \rangle \\ &= -\frac{1}{8} \langle\langle \Omega | \kappa^2 H_{ij} H_{k\ell} | \Omega \rangle\rangle \times \frac{1}{A^2} \langle \Omega | \kappa^2 \partial_\ell H_{im} \partial_j H_{km} | \Omega \rangle + O(\kappa^6) , \end{aligned} \quad (132)$$

$$= -\frac{1}{10} \left[\delta_{i(k} \delta_{\ell)j} - \frac{\delta_{ij} \delta_{k\ell}}{3} \right] x \times \frac{3}{10} [\delta_{i(j} \delta_{\ell)k} - 2\delta_{ik} \delta_{j\ell}] H\dot{x} + O(H\dot{x}x^2) , \quad (133)$$

$$= H\dot{x} \left\{ 0 + \frac{9}{40} x + O(x^2) \right\} . \quad (134)$$

Substituting expansions (125), (131) and (134) in our equation (67) for \dot{A}/A gives:

$$\frac{\dot{A}^2}{A^2} = H^2 + \frac{1}{3} \delta\Lambda_1 \kappa^2 H^4 - H\dot{x} \left\{ \frac{1}{2} + \frac{2}{5} x + O(x^2) \right\} . \quad (135)$$

Taking the square root and retaining only leading logarithm terms gives:

$$\frac{\dot{A}}{A} = H + \frac{1}{6} \delta\Lambda_1 \kappa^2 H^3 - \dot{x} \left\{ \frac{1}{4} + \frac{1}{5} x + O(x^2) \right\} . \quad (136)$$

Integrating reveals corrections to the effective scale factor as a series in powers of the parameter (124):

$$\ln[A(t)] = \ln(a) + \frac{2\pi^2}{3} \delta\Lambda_1 x - \frac{1}{4} x - \frac{1}{10} x^2 + O(x^3) , \quad (137)$$

where we remind ourselves that:

$$x(t) \equiv \frac{\kappa^2 H^2}{4\pi^2} \ln(a) . \quad (138)$$

One might think that relation (137) reflects an unfortunate background dependence of our formalism. In the sense that the equation was derived using perturbation theory it certainly does depend upon the zeroth order result. However, the larger issue is whether or not it represents the perturbative expansion of a background independent result. For proper consideration of this matter it is crucial to distinguish dependence upon the initial state from dependence upon the background. The leading logarithm approximation, and our entire physical picture, only applies to an initial state which suffers a long period of inflation. This is a wide class but not a universal one. Many initial states are so heavily loaded with gravitons that they suffer gravitational collapse before beginning to inflate. We have nothing to say about such states; our focus is instead on states which are initial empty enough that inflation begins. The classical evolution of such a state is to locally approach the de Sitter geometry we used as the background in (4). In that case there is a perfectly background independent way of interpreting the factors of $\ln(a) = Ht$ in expression (137): H^2 is the invariant acceleration measured by geodesic deviation and t is the invariant time from the point of observation to the initial value surface.

5 Invariant Acceleration Observable

Equation (137) shows that the effective scale factor experiences secular slowing at 2-loop order. However, this is not enough to conclude that observers actually experience slowing, quite apart from the validity of the Effective Scale Factor Approximation. The problem concerns the noncommutativity of two operations:

- Forming an invariant measure of expansion; and
- Taking the expectation value.

The variable $A(t)$ can be regarded as the one third power of the local volume factor in synchronous gauge. Had the metric been classical, the logarithmic time derivative of $A(t)$ would indeed give the expansion rate. But Unruh has shown that one cannot infer physics this way from the expectation value of the metric [43]. Instead of taking the expectation value and then forming an invariant, the correct procedure is form an invariant observable from the quantum metric and then take its expectation value. Models of scalar-driven

inflation which seemed to show secular slowing at 1-loop order from the expectation value of the metric [44, 45] show no such 1-loop effect when examined with an invariant expansion operator [46, 47].

Finding an invariant measure of the expansion rate in pure quantum gravity is more difficult than for scalar-driven inflation because one lacks the preferred coordinate system in which the scalar is homogeneous. A reasonable proposal is based upon using the equation of geodesic deviation to measure the local acceleration between objects released from rest at some point x^μ [21]. In the synchronous gauge we have been using this quantity takes the form:

$$\gamma(x) = \frac{1}{g_{rs} \Delta^r \Delta^s} \left(\frac{1}{2} \ddot{g}_{ij} - \frac{1}{4} g^{k\ell} \dot{g}_{ik} \dot{g}_{j\ell} \right) \Delta^i \Delta^j , \quad (139)$$

where Δ^i is the spacelike separation of two initially parallel timelike geodesics.

To make $\gamma(x)$ a full invariant one needs to average over the initial separation vector Δ^i and also the position x^μ . A \mathbb{C} -number such as Δ^i cannot transform as a vector because only the fields of a quantum field theory transform. We therefore make use of an old trick [48] to convert it into a local Lorentz vector using the vierbein field $e^i_a(x)$:

$$\Delta^i \longrightarrow e^i_c(x) \Delta^c . \quad (140)$$

It will be seen that this takes $\gamma(x)$ to the form:

$$\gamma(x) = \left(\frac{1}{2} \ddot{g}_{ij} - \frac{1}{4} g^{kl} \dot{g}_{ik} \dot{g}_{jl} \right) e^i_b e^j_c \hat{n}^b \hat{n}^c , \quad (141)$$

where \hat{n} is the \mathbb{C} -number unit vector in the initial separation direction:

$$\hat{n}^b \equiv \frac{\Delta^b}{\sqrt{\Delta^c \Delta^c}} . \quad (142)$$

We can now take the \mathbb{C} -number average over directions using

$$\int d^{D-2} \hat{n} \hat{n}^b \hat{n}^c = \frac{\delta^{bc}}{D-1} , \quad (143)$$

to attain a scalar quantity:

$$\bar{\gamma}(x) \equiv \int d^{D-2} \hat{n} \gamma(x) = \frac{g^{ij} \ddot{g}_{ij}}{2(D-1)} - \frac{g^{ij} g^{kl} \dot{g}_{ik} \dot{g}_{jl}}{4(D-1)} . \quad (144)$$

To achieve a full invariant we must multiply by $\sqrt{-g(x)}$ and integrate. Because the universe was released in a prepared state at $t = 0$, we can weight this integral by an arbitrary function of the invariant time from the initial value surface. That time is not an operator at all in synchronous gauge, it is just the coordinate time t . Moreover, we may as well choose the weight to be a delta function selecting a particular time. That still leaves the integration over space at this time. However, because the initial state is homogeneous, as is the gauge, we can dispense with even this step, although we do still need to multiply by $\sqrt{-g(t, \vec{x})}$. The same procedure has already been exploited in computing the 1-loop expectation value of an invariant 2-point function in flat space background [48].

We now make the Effective Scale Factor Approximation laid out in relations (43-45).⁷ We first evaluate $\bar{\gamma}$ with the substitution $g_{ij} = A^2 \bar{g}_{ij}$, then take account of simplifications resulting from the tracelessness of H_{ij} :

$$\bar{\gamma} \longrightarrow \frac{\dot{A}^2}{A^2} + \frac{d}{dt} \frac{\dot{A}}{A} + \frac{\dot{A}}{A} \frac{\bar{g}^{ij} \dot{\bar{g}}_{ij}}{D-1} + \frac{\bar{g}^{ij} \ddot{\bar{g}}_{ij}}{2(D-1)} - \frac{\bar{g}^{ij} \bar{g}^{k\ell} \dot{\bar{g}}_{ik} \dot{\bar{g}}_{j\ell}}{4(D-1)} , \quad (145)$$

$$= \frac{\dot{A}^2}{A^2} + \frac{d}{dt} \frac{\dot{A}}{A} + \frac{\bar{g}^{ij} \bar{g}^{k\ell} \dot{\bar{g}}_{ik} \dot{\bar{g}}_{j\ell}}{4(D-1)} . \quad (146)$$

Let us now take note of the highly significant fact that the measure factor is a \mathbb{C} -number in the Effective Scale Factor Approximation:

$$\sqrt{-g(t, \vec{x})} \longrightarrow [A(t)]^D . \quad (147)$$

We can therefore dispense with it altogether in the expectation value:

$$\begin{aligned} \langle\langle \Omega | \bar{\gamma} | \Omega \rangle\rangle &= \frac{\dot{A}^2}{A^2} + \frac{d}{dt} \frac{\dot{A}}{A} + \frac{1}{4(D-1)} \langle\langle \Omega | B[\kappa H]_{ijk\ell} B[\kappa H]_{jipq} | \Omega \rangle\rangle \\ &\quad \times \langle \Omega | \kappa^2 \dot{H}_{k\ell} \dot{H}_{pq} | \Omega \rangle , \end{aligned} \quad (148)$$

where we have also used relation (64).

⁷Note that this entails the potentially invalid step of treating a part of the average geometry — the effective scale factor $A(t)$ — as though it is the quantum geometry. The fully correct procedure is to form an invariant operator first and then take its VEV. Our use of the Effective Scale Factor Approximation will only be justified if one can derive the approximation at leading logarithm order. We feel this can be done but we acknowledge that it has not been done yet.

At this stage we substitute equation (67) for the effective scale factor in (148):

$$\begin{aligned}
\langle\langle \Omega | \bar{\gamma} | \Omega \rangle\rangle &= H^2 + \frac{d}{dt} \frac{\dot{A}}{A} + \frac{\delta\Lambda_1 \kappa^2 H^4}{D-1} \\
&+ \frac{1}{4(D-2)} \langle\langle \Omega | B[\kappa H]_{ijkl} B[\kappa H]_{jipq} | \Omega \rangle\rangle \times \langle \Omega | \kappa^2 \dot{H}_{kl} \dot{H}_{pq} | \Omega \rangle \\
&+ \frac{1}{(D-2)(D-1)} \langle\langle \Omega | \frac{3}{4} B[\kappa H]_{ijkl} B[\kappa H]_{jipq} \bar{g}^{mn} \\
&\quad - B[\kappa H]_{nikl} B[\kappa H]_{mjipq} \bar{g}^{ij} | \Omega \rangle\rangle \times \frac{1}{A^2} \langle \Omega | \kappa^2 H_{kl,m} H_{pq,n} | \Omega \rangle . \quad (149)
\end{aligned}$$

We have encountered each of the three terms on the right hand side (with slightly different numerical coefficients) in the equation for the effective scale factor; taking $D = 4$ and substituting expansions (125), (131) and (134) gives:

$$\langle\langle \Omega | \bar{\gamma} | \Omega \rangle\rangle = H^2 + \frac{d}{dt} \frac{\dot{A}}{A} + \frac{1}{3} \delta\Lambda_1 \kappa^2 H^4 - \frac{3}{8} H \dot{x} \{ 1 + x + O(x^2) \} . \quad (150)$$

From expression (136) we see that the time derivative of \dot{A}/A is always subdominant, and we should choose $\delta\Lambda_1$ so that initially the observable equals to H^2 :

$$\langle\langle \Omega | \bar{\gamma}(t=0) | \Omega \rangle\rangle = H^2 \quad \implies \quad \delta\Lambda_1 = \frac{9}{32\pi^2} . \quad (151)$$

Our final result is:

$$\langle\langle \Omega | \bar{\gamma} | \Omega \rangle\rangle = H^2 \left\{ 1 - \frac{3}{8} \left(\frac{\kappa H}{2\pi} \right)^4 \ln(a) + O(\kappa^6 H^6 \ln^2(a)) \right\} . \quad (152)$$

Hence, the observable $\bar{\gamma}$ measures a slowdown of the expansion rate.

6 Epilogue

During inflation quantum gravitation loop corrections are enhanced by factors of the number of e-foldings since inflation began. These enhancement factors are known as *infrared logarithms* and there can be at most one such factor for each extra power of GH^2 in perturbation theory. The set of terms

for which this maximum number is attained is known as the *leading logarithm approximation*.

Quantum gravity is vastly better behaved in the leading logarithm approximation than in general. The reason for this is that, to reach leading logarithm order, every undifferentiated free field must contribute to an infrared logarithm, which precludes these fields from producing ultraviolet divergences. From the plethora of BPHZ counterterms which permeate full quantum gravity, only the 1-loop renormalization of the cosmological constant contributes at leading logarithm order.

We have explored a dynamical assumption called the *Effective Scale Factor Approximation*. Under this assumption the graviton remains transverse-traceless and free, but propagates in the background geometry of an effective scale factor $A(t)$ which is determined from the expectation value of the g_{00} equation. By extending the formalism to the first subdominant logarithm order one can even describe cosmological perturbations of the correct strength.

Under the Effective Scale Factor Approximation all perturbative computations are straightforward. We obtain explicit 1- and 2-loop results for the effective scale factor and for an invariant observable based on using the equation of geodesic deviation to probe the local acceleration. It is even possible to give a nonperturbative, numerical formulation to quantum gravity within the context of the Effective Scale Factor Approximation.

The Effective Scale Factor Approximation may not be entirely correct but it does illustrate how simple inflationary quantum gravity can become at leading logarithm order. It also serves as a serious test of the “null hypothesis” that there are no significant corrections from infrared gravitons. For if the hypothesis was correct, then either the effective scale factor $A(t)$ would receive no corrections or else they would drop out when evaluating the expectation value of the invariant acceleration observable. However, neither of these possibilities occurs.

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References

- [1] N. C. Tsamis and R. P. Woodard, Nucl. Phys. **B474** (1996) 235, hep-ph/9602315; Annals Phys. **253** (1997) 1, hep-ph/9602316; Phys. Rev. **D54** (1996) 2621, hep-ph/9602317.
- [2] S. P. Miao and R. P. Woodard, Class. Quant. Grav. **23** (2006) 1721, gr-qc/0511140; Phys. Rev. **D74** (2006) 024021, gr-qc/0603135; Class. Quant. Grav. **25** (2008) 145009, arXiv:0803.2377; S. P. Miao, “The Quantum-Corrected Fermion Mode Function During Inflation,” arXiv:0705.0767.
- [3] S. Weinberg, Phys. Rev. **D74** (2006) 023508, hep-th/0605244.
- [4] S. Weinberg, Phys. Rev. **D72** (2005) 043514, hep-th/0506236; K. Chaicherdsakul, Phys. Rev. **D75** (2007) 063522, hep-th/0611352.
- [5] D. Boyanovsky, H. J. de Vega and N. G. Sanchez, Nucl. Phys. **B747** (2006) 25, astro-ph/0503669; Phys. Rev. **D72** (2005) 103006, astro-ph/0507596; M. Sloth, Nucl. Phys. **B748** (2006) 149, astro-ph/0604488; A. Bilandžić and T. Prokopec, Phys. Rev. **D76** (2007) 103507, arXiv:0704.1905; M. van der Meulen and J. Smit, JCAP **0711** (2007) 023, arXiv:0707.0842; Y. Urakawa and K. I Maeda, arXiv:0801.0126.
- [6] N. C. Tsamis and R. P. Woodard, Phys. Lett. **B426** (1998) 21, hep-ph/9710466; V. K. Onemli and R. P. Woodard, Class. Quant. Grav. **19** (2002) 4607, gr-qc/0204065; Phys. Rev. **D70** (2004) 107301, gr-qc/0406098; T. Brunier, V. K. Onemli and R. P. Woodard, Class. Quant. Grav. **22** (2005) 59, gr-qc/0408080; E. O. Kahya and V. K. Onemli, Phys. Rev. **D76** (2007) 043512, gr-qc/0612026.
- [7] T. Prokopec, O. Tornkvist and R. P. Woodard, Phys. Rev. Lett. **89** (2002) 101301, astro-ph/0205331; Ann. Phys. **303** (2003) 251, gr-qc/0205130; T. Prokopec and R. P. Woodard, Ann. Phys. **312** (2004) 1, gr-qc/0310056; T. Prokopec and E. Puchwein, JCAP **0404** (2004) 007, astro-ph/0312274; T. Prokopec, N. C. Tsamis and R. P. Woodard, Class. Quant. Grav. **24** (2007) 201, gr-qc/0607094; arXiv:0802.3673.

- [8] T. Prokopec and R. P. Woodard, JHEP **0310** (2003) 059, astro-ph/0309593; B. Garbrecht and T. Prokopec, Phys. Rev. **D73** (2006) 064036, gr-qc/0602011.
- [9] S. P. Miao and R. P. Woodard, Phys. Rev. **D74** (2006) 044019, gr-qc/0602110.
- [10] E. Komatsu et al., Astrophys. J. Suppl. **180** (2009) 330, arXiv:0803.0547.
- [11] A. A. Starobinskiĭ, “Stochastic de Sitter (inflationary) Stage in the Early Universe,” in *Field Theory, Quantum Gravity and Strings*, ed. H. J. de Vega and N. Sanchez (Springer-Verlag, Berlin, 1986) pp. 107-126.
- [12] R. P. Woodard, Nucl. Phys. Proc. Suppl. **148** (2005) 108, astro-ph/0502556; N. C. Tsamis and R. P. Woodard, Nucl. Phys. **B724** (2005) 295, gr-qc/0505115.
- [13] A. A. Starobinskiĭ and J. Yokoyama, Phys. Rev. **D50** (1994) 6357, astro-ph/9407016.
- [14] T. Prokopec, N.C. Tsamis and R. P. Woodard, Ann. Phys. **323** (2008) 1324, arXiv:0707.0847.
- [15] A. M. Polyakov, Nucl. Phys. **B797** (2008) 199, arXiv:0709.2899.
- [16] I. Antoniadis, P. O. Mazur and E. Mottola, New J. Phys. **9** (2007) 11, gr-qc/0612068.
- [17] N. C. Tsamis and R. P. Woodard, Phys. Lett. **B301** 1993, 351.
- [18] N. C. Tsamis and R. P. Woodard, Ann. Phys. **267** (1998) 145, hep-ph/9712331; R. P. Woodard, Nucl. Phys. Proc. Suppl. **104** (2002) 173, gr-qc/0107042.
- [19] J Garriga and T. Tanaka, Phys. Rev. **D77** (2008) 024021, arXiv:0706.0295.
- [20] N. C. Tsamis and R. P. Woodard, Phys. Rev. **D78** (2008) 028501, arXiv:0708.2004.

- [21] N. C. Tsamis and R. P. Woodard, *Class. Quant. Grav.* **22** (2005) 4171, gr-qc/0506089.
- [22] F. Bloch and H. Nordsieck, *Phys. Rev.* **52** (1937) 54.
- [23] S. Weinberg, *Phys. Rev.* **140** (1965) B516.
- [24] G. Feinberg and J. Sucher, *Phys. Rev.* **166** (1968) 1638; S. D. H. Hsu and P. Sikivie, *Phys. Rev.* **D49** (1994) 4951, hep-ph/9211301.
- [25] D. M. Capper, M. J. Duff and L. Halperin, *Phys. Rev.* **D10** (1974) 461; D. M. Capper and M. J. Duff, *Nucl. Phys.* **B84** (1974) 147; D. M. Capper, *Nuovo Cimento* **A25** (1975) 29; M. J. Duff and J. T. Liu, *Phys. Rev. Lett.* **85** (2000) 2052, hep-th/0003237.
- [26] J. F. Donoghue, *Phys. Rev. Lett.* **72** (1994) 2996, gr-qc/9310024; *Phys. Rev.* **D50** (1994) 3874, gr-qc/9405057.
- [27] I. J. Muzinich and S. Kokos, *Phys. Rev.* **D52** (1995) 3472, hep-th/9501083; H. Hamber and S. Liu, *Phys. Lett.* **B357** (1995) 51, hep-th/9505182; A. Akhundov, S. Belucci and A. Shiekh, *Phys. Lett.* **B395** (1998), gr-qc/9611018; I. B. Kriplovich and G. G. Kirilin, *J. Exp. Theor. Phys.* **98** (2004) 1063, gr-qc/0402018; I. B. Kriplovich and G. G. Kirilin, *J. Exp. Theor. Phys.* **95** (2002) 981, gr-qc/0207118.
- [28] L. P. Grishchuk, *Sov. Phys. JETP* **40** (1975) 409.
- [29] N. C. Tsamis and R. P. Woodard, *Phys. Lett.* **B292** (1992) 269.
- [30] L. H. Ford and L. Parker, *Phys. Rev.* **D16** (1977) 245.
- [31] A. Vilenkin, *Nucl. Phys.* **B226** (1983) 527.
- [32] N. C. Tsamis and R. P. Woodard, *Class. Quant. Grav.* **11** (1994) 2969.
- [33] M. Musso, “A New Diagrammatic Representation for Correlation Functions in the In-In Formalism,” hep-th/0611258.
- [34] A. Vilenkin and L. H. Ford, *Phys. Rev.* **D26** (1982) 1231; A. D. Linde, *Phys. Lett.* **116B** (1982) 335; A. A. Starobinskiĭ, *Phys. Lett.* **117B** (1982) 175.

- [35] N. C. Tsamis and R. P. Woodard, *Ann. Phys.* **238** (1995) 1.
- [36] N. C. Tsamis and R. P. Woodard, *Commun. Math. Phys.* **162** (1994) 217.
- [37] N. C. Tsamis and R. P. Woodard, *Class. Quant. Grav.* **20** (2003) 5205, astro-ph/0206010; *Class. Quant. Grav.* **21** 93, astro-ph/0306602; *Phys. Rev.* **D69** (2004) 084005; astro-ph/0307463.
- [38] A. S. Goncharov, A. D. Linde and V. F. Mukhanov, *Int. J. Mod. Phys.* **A2** (1987) 561; A. D. Linde and A. Mezhlumian, *Phys. Lett.* **B307** (1993) 25, gr-qc/9304015.
- [39] T. M. Janssen, S. P. Miao, T. Prokopec and R. P. Woodard, *Class. Quant. Grav.* **25** (2008) 245013, arXiv:0808.2449; T. Janssen, S. P. Miao and T. Prokopec, “Graviton one-loop effective action and inflationary dynamics,” arXiv:0807.0439; T. Janssen and T. Prokopec, “Implications of the graviton one-loop effective action on the dynamics of the Universe,” arXiv:0807.0447; “A graviton propagator for inflation,” arXiv:0707.3919.
- [40] N. C. Tsamis and R. P. Woodard, *Ann. Phys.* **321** (2006) 875, gr-qc/0506056.
- [41] F. Finelli, G. Marozzi, G. P. Vacca and G. Venturi, *Phys. Rev.* **D71** (2005) 023522, gr-qc/0407101.
- [42] L. H. Ford, *Phys. Rev.* **D31** (1985) 710.
- [43] W. Unruh, “Cosmological Long Wavelength Perturbations,” astro-ph/9802323.
- [44] V. F. Mukhanov, L. R. Abramo and R. H. Brandenberger, *Phys. Rev. Lett.* **78** (1997) 1624, gr-qc/9609026; L. R. Abramo, R. H. Brandenberger and L. R. Abramo, *Phys. Rev.* **D56** (1997) 3248, gr-qc/9704037.
- [45] L. R. Abramo and R. P. Woodard, *Phys. Rev.* **D60** (1999) 044010, astro-ph/9811430.
- [46] L. R. Abramo and R. P. Woodard, *Phys. Rev.* **D65** (2002) 043507, astro-ph/0109271; *Phys. Rev.* **D65** (2002) 063515, astro-ph/0109272; *Phys. Rev.* **D65** (2002) 063516; astro-ph/0109273.

- [47] G. Geshnizjani and R. Brandenberger, Phys. Rev. **D66** (2002) 123507, gr-qc/0204074.
- [48] R. P. Woodard, Phys. Lett. **B148** (1984) 440; N. C. Tsamis and R. P. Woodard, Annals Phys. **215** (1992) 96.